(a) (14 points) **Short Answer:** Give short and precise answer for each of the questions below only on the space provided.

1. The Value of limit
   (a) $\lim_{x \to 1} \frac{x^{100} - 1}{x - 1} =$ __________
   (b) $\lim_{x \to 0} (e^x + x)^{\frac{1}{x}} =$ __________
   (c) $\lim_{x \to \infty} x^{(\frac{1}{x})} =$ __________
2. If the functions
   $$g(x) = \begin{cases} be^x + a + 1, x \leq 0 \\ ax^2 + b(x+3), 0 < x \leq 1 \\ a\cos(\pi x) + 7bx, x > 1 \end{cases}$$
   is continuous everywhere, the values of $a =$ __________ and $b =$ __________
3. Equation of the normal line to the parabola $y = x^2 + 4x - 3$ that is parallel to the line $x - 2y = 7$ is __________
4. $\frac{d}{dx}\left(\tan\left(\sqrt{\cosh^{-1}(x)}\right)\right) =$ __________
5. The value of the integral
   (a) $\int \frac{x+1}{x^3 + x}\, dx =$ __________
   (b) $\int_4^\infty \frac{1}{(x-3)^{\frac{5}{3}}} dx =$ __________
   (c) $\int x \ln(x^4 + 1)\, dx =$ __________
6. (a) State the Fundamental Theorem of Calculus Part I (FTCI)
   ____
   ____
   ____
   (b) $G(x) = \int_{e^{-x}}^{e^x} \ln(t)\, dt$ then using (a) above $G'(x) =$ __________
7. The area of the region enclosed by the line $y = x$ and the curve $y = x^3$ is __________
8. The Volume of the solid generated by revolving the region bounded by $x = y^2, x =$ and $y = 0$ about x-axis is __________

(b) (26 points) **Workout Problems:** Show all the necessary steps clearly and neatly on the space provided.

1. (6 points) Use $\epsilon - \delta$ definition to show that $\lim_{x \to 0^+} \frac{x+1}{1+\sqrt{x}} = 1$

2. (6 points) Sand is flowing from a pipe at the constant rate of $45m^3/s$, and is falling **in a conical** pile. The radius of this pile is always 1.5 times the altitude (height). **Find** the rate at which the altitude of the pile is increasing when the altitude is $2m$.

3. **(7 points)** A closed rectangular container with square base is to have a volume of $5m^3$. Suppose the material for the two square faces will cost 5 birr per square meter, the material for the rest four rectangular faces will be 8 birr per square meter. Find the dimensions of the container that minimize the cost.

4. (7 points) Let $f(x) = \frac{e^x}{1-e^x}$. Then find

(a) The intercepts

(b) The asymptotes

(c) The interval on which f is increasing and on which it is decreasing

(d) The relative extreme value of f

(e) Concavity and inflection points

(f) Sketch the graph of f

Sol:

a. has no x & y intercept

b. ~~V.A = e^x~~ V.A $x=0$

c. $f'(x)$

$$\frac{e^x}{1-e^x} = \frac{e^x(1-e^x) - (-e^x)e^x}{(1-e^x)^2}$$

$$= e^x - e^{x^2} + e^{x^2}$$